Investigating the Collapse and Convergence of Particle-Wave Statistics in Pilot-Wave Hydrodynamics

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Skyler Mao
Mentor: Dave Darrow

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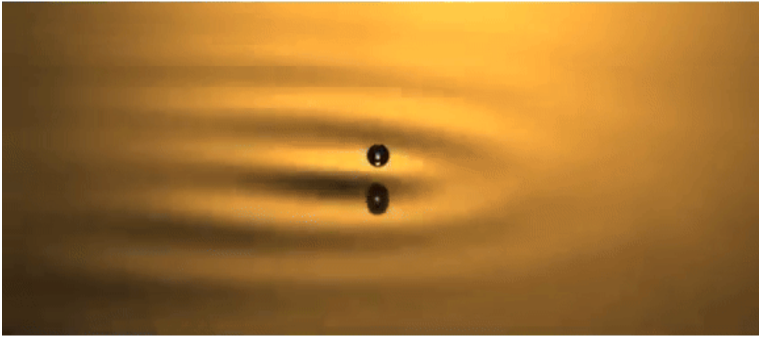
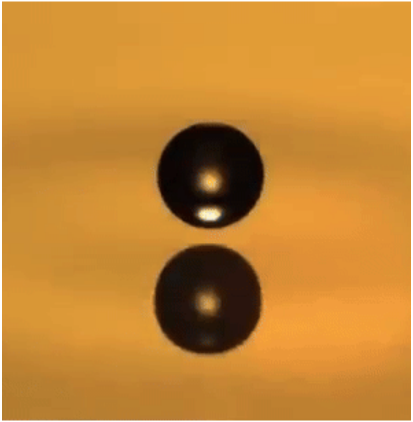
The Walking Droplet System

In the walking droplet system of Couder and Fort, a millimetric oil droplet bounces on a vibrating fluid surface, propelled forward by its self-generated waves.



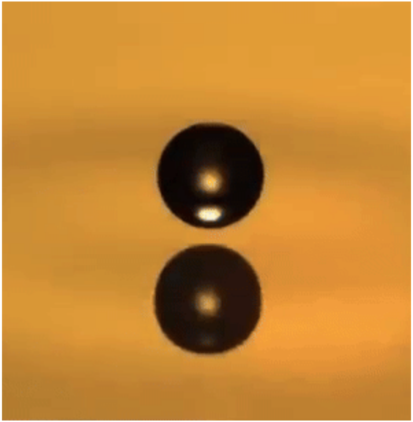
How the Droplet Walks

- Surface tension and the fast vibration allows the droplet to bounce on the fluid bath



How the Droplet Walks

- Surface tension and the fast vibration allows the droplet to bounce on the fluid bath
- The vibration of the fluid bath allows the droplet to achieve “resonance” with its wave field, propelling itself forward



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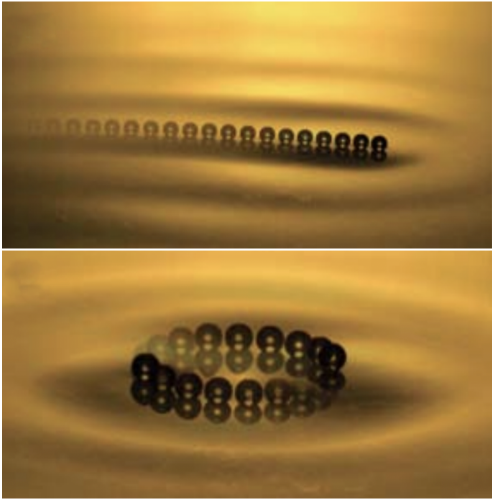
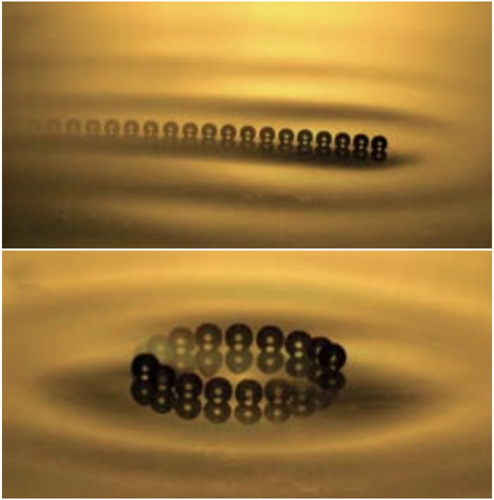


- After strobing, the droplet appears to “ride” the wave field

How the Droplet Walks



- After strobing, the droplet appears to “ride” the wave field
- The droplet’s dynamics can be chaotic or converge to a regular pattern

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A Quantum Analogue

- Due to the coupled dynamics of the wave-particle system, the droplet exhibits statistical behavior qualitatively similar to that of quantum particles

A Quantum Analogue

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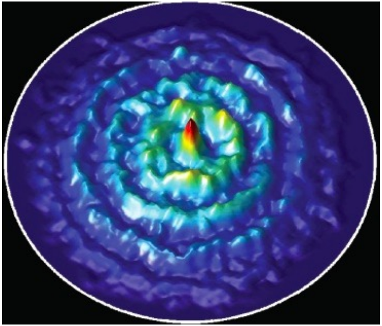
- This classical pilot-wave theory is reminiscent of several earlier quantum pilot-wave theories, such as those of de Broglie and Bohm

Example 1: Quantum Corral

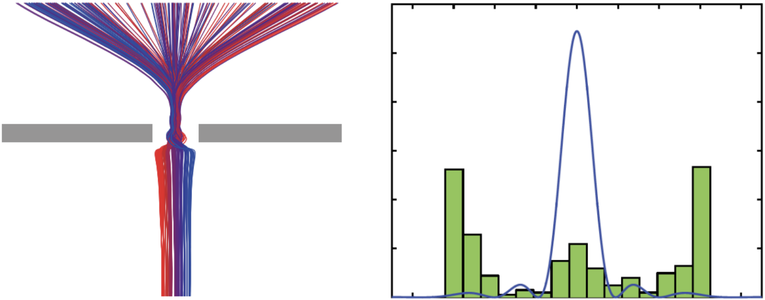
The droplet statistics in a circular bath resemble that of a quantum corral.



(Harris et al. 2013)

Example 2: Single Slit Diffraction

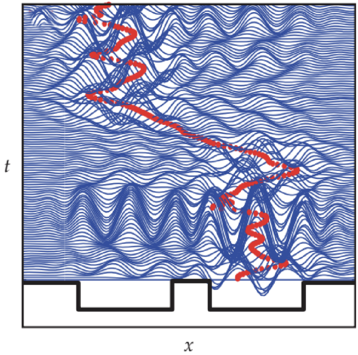
When the droplet passes through a slit formed by barriers, it displays statistics similar to single-slit diffraction.



(Pucci et al. 2017)

Example 3: Droplet Tunneling

The droplet can "tunnel" past submerged barriers in the fluid bath.



(Nachbin et al. 2017)

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PDF and MWF

Over time, the droplet's position converges to a probability density function (PDF), and the time-average of the underlying wave converges to a mean wave field (MWF).

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In quantum mechanics, the closest analogy to the MWF is the particle's wavefunction, and the closest analogy to the PDF is the wavefunction squared (i.e. the probability distribution of the particle).

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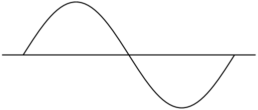
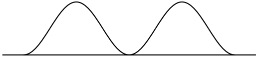
Inspired by this, we ask: does such a correspondence occur in the walking droplet system?





Durey's Theorem

A theorem of Durey, Milewski, and Bush (2018) relates the PDF to the MWF in a particular droplet model and in certain 2D geometries:

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Theorem (DMB 2018)

Suppose a droplet follows the 2D equations of Molaçek and Bush (2013), either in a free system or a circular corral.

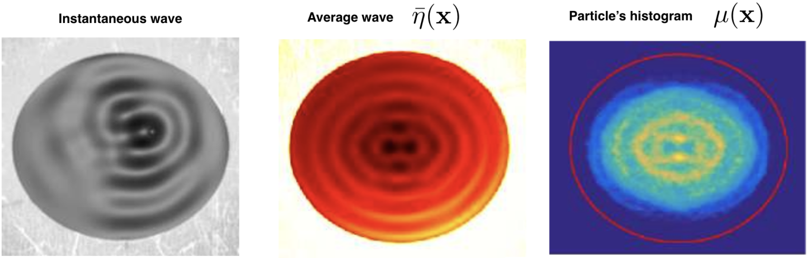
Assuming there exists a stationary probability distribution $\mu(x)$ for the droplet position and that the system dynamics are ergodic, then the mean wave field $\bar{\eta}(x)$ satisfies

$$\bar{\eta}(x) = \int_{\mathbb{R}^2} \eta_B(x - y)\mu(y)\, dy,$$

where $\bar{\eta}_B(x)$ is the wave field of a stationary bouncer at the origin.

Can We Do Better?

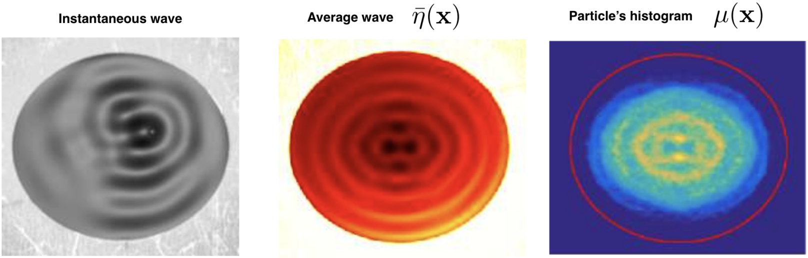
- Similar statistical behavior has been observed in other geometries (e.g. in an elliptical corral)



(Bush et al.)

Can We Do Better?

- Similar statistical behavior has been observed in other geometries (e.g. in an elliptical corral)
- Can we extend Durey's result to *any* walking droplet system?



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Relating the PDF and MWF

We expand on Durey's result and derive a general relationship between the PDF and MWF.

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Theorem (Relating PDF and MWF, informal)

For (almost) any walking droplet system, the MWF is the sum of the MWFs of a fixed, bouncing droplet at each point multiplied by the probability density at that point.

Relating the PDF and MWF

Theorem (Relating PDF and MWF)

Suppose $X(t) \in \mathbb{C}^m$ is piecewise constant on $[k, k+1)$, for $k \in \mathbb{N}$, and undergoes an ergodic process that converges to a probability density μ on $\mathbb{C}^m$. Suppose that $y(t) \in \mathbb{C}^n$ solves

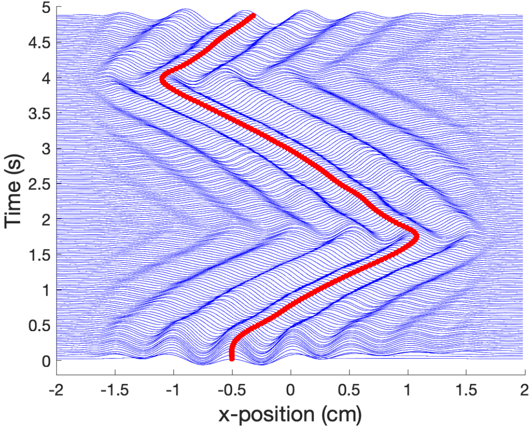
$$\dot{y} = L(t)y + b(t, X(t)),$$

where $L(t) \in \mathbb{C}^{n\times n}$ is 1/2-periodic and negative definite, and $b(t, x)$ is 1-periodic in t for each fixed value $x \in \mathbb{C}^m$. Then, the mean field $\langle y \rangle \doteq \lim_{N\to\infty} N^{-1} \sum_{t=1}^{N} y(t)$ is given by

$$\langle y \rangle = \int_{\mathbb{R}} \mu(\mathcal{X}) y_0(x, \mathcal{X}) d\mathcal{X},$$

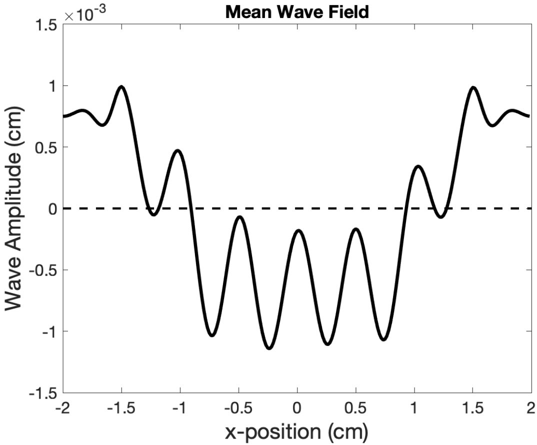
where $y_0(x, \mathcal{X})$ is the mean field when $X(t) = \mathcal{X}$ is constant.

Numerical Simulation

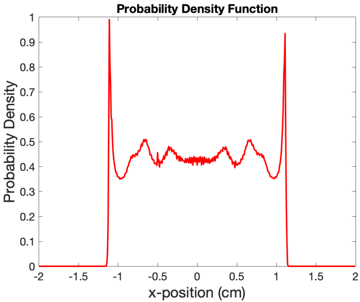


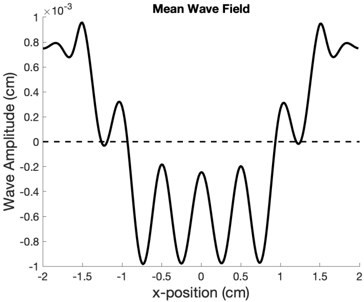
Droplet traversing a 3cm-width domain with $\Gamma/\Gamma_F = 0.85$

Numerical Simulation



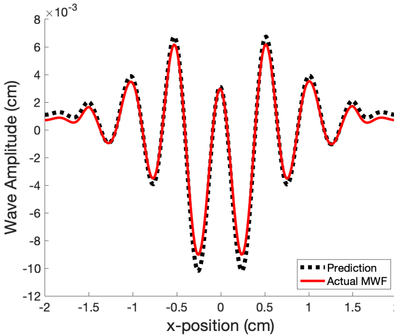
Examples of PDF and MWF

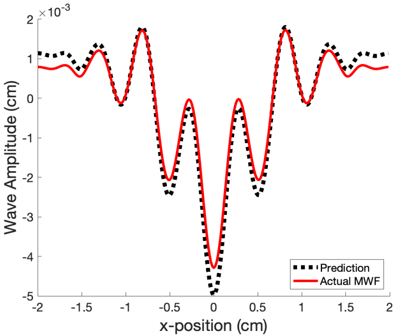




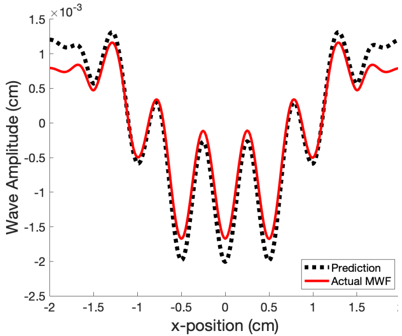
Over a long time (e.g. $\approx 10^4$ bounces), we retrieve the PDF and MWF.

Numerical Results: $\Gamma/\Gamma_F = 0.70$ and 0.75, Width $= 3$ cm

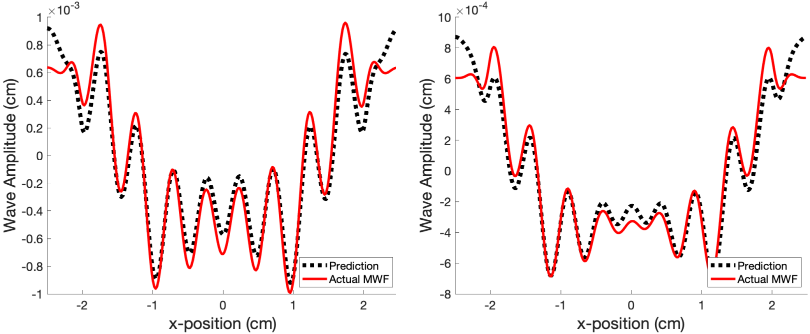




Numerical Results: $\Gamma/\Gamma_F = 0.80$ and 0.85, Width $= 3$ cm



Numerical Results: $\Gamma = 0.70$, $\Gamma = 0.75$, Width $= 4$ cm



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Measurement

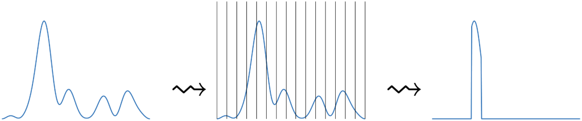


Figure: Quantum mechanics: A particle's wavefunction collapse.

Measurement

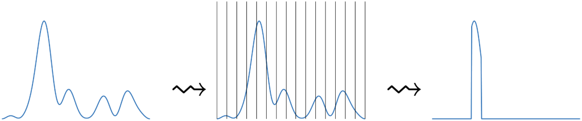


Figure: Quantum mechanics: A particle's wavefunction collapse.

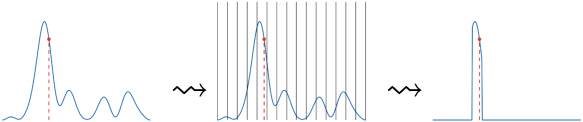
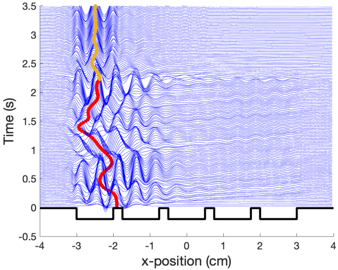
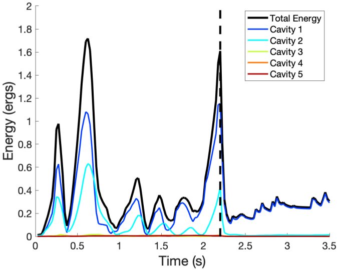


Figure: Walking droplet system: The particle always had a fixed position x_p, and x_p determines which interval the wave height collapses into.

Measurement

We perform a “measurement” in the walking droplet system by imposing cavities on the domain.





Measurement

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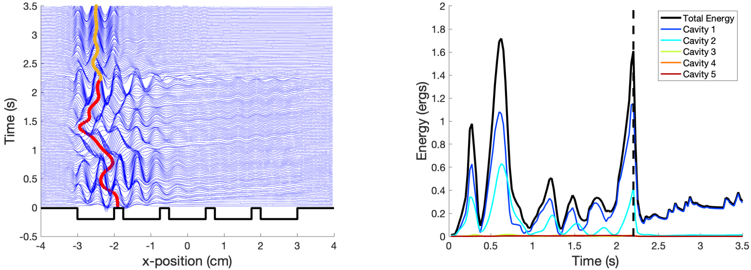

Measurement with 5 cavities in a 6 cm domain.

Future Directions with Measurement

- How are the PDF and MWF impacted by imposing cavities?

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- How does this extend to a 2-droplet system?

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- How are the PDF and MWF impacted by imposing cavities?
- How does this extend to a 2-droplet system?
- Can we use similar techniques to measure the rate of which these fields converge after imposing and removing the measurement apparatus?

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- I would like to thank my mentor Dave Darrow for his invaluable support and guidance throughout this research project.

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Thank you!